Känguru der Mathematik 2016
Level Junior (Grade 9 and 10)
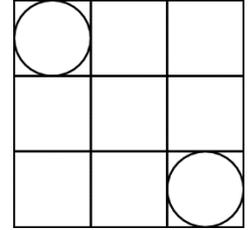# Österreich – 17.03.2016

- 3 Points Questions -

1. The arithmetic mean of four numbers is 9. What is the fourth number if the three other numbers are 5, 9 and 12?

(A) 6   (B) 8   (C) 9   (D) 10   (E) 36

2. Which of the following numbers is closest to the number $\frac{17 \times 0.3 \times 20.16}{999}$?

(A) 0.01   (B) 0.1   (C) 1   (D) 10   (E) 100

3. Ruth takes part in the kangaroo competition where 30 questions have to be answered. She answers every question and each answer is either right or wrong. She has 50% more right than wrong answers. How many of her answers are right?

(A) 10   (B) 12   (C) 15   (D) 18   (E) 20

4. Five points are given in a Cartesian coordinate system: P(-1, 3), Q(0, -4), R(-2, -1), S(1, 1), T(3, -2). Four of these five points are vertices of a square. Which point does not belong there?

(A) P   (B) Q   (C) R   (D) S   (E) T

5. If a positive whole number x is divided by 6, the remainder is 3. What is the remainder if $3 \times x$ is divided by 6?

(A) 4   (B) 3   (C) 2   (D) 1   (E) 0

6. 2016 hours are how many weeks?

(A) 6   (B) 8   (C) 10   (D) 12   (E) 16

7. Lukas invents his own notation for negative numbers. When counting backwards he writes:
... 3, 2, 1, 0, 00, 000, 0000, ... What is the result of the calculation 000 + 0000 in his notation?

(A) 1   (B) 00000   (C) 000000   (D) 0000000   (E) 00000000

8. I have some unusual dice. On their faces are the digits 1 to 6 as usual, however the odd numbers are negative (so -1, -3, -5 instead of 1, 3, 5). I throw two such dice at the same time. Which of the following sums can I definitely not achieve with one such throw?

(A) 3   (B) 4   (C) 5   (D) 7   (E) 8

9. Step by step the word VELO is changed into the word LOVE. In every step two adjacent letters are allowed to be swapped around. What is the minimum amount of steps needed?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

10. Sven writes five different single-digit positive whole numbers on a board. He realises that no sum of two of these numbers is equal to 10. Which of the following numbers has Sven definitely written on the board?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

- 4 Points Questions -

11. For the real numbers a, b, c, d the following holds true: $a + 5 = b^2 - 1 = c^2 + 3 = d - 4$. Which of the numbers a, b, c, d is biggest?

(A) a (B) b (C) c (D) d (E) It cannot be uniquely determined using this information.

12. A 3×3 field is made up of 9 unit squares. In two of these squares, circles are inscribed as shown in the diagram.
How big is the shortest distance between these circles?

(A) $2\sqrt{2} - 1$ (B) $\sqrt{2+1}$ (C) $2\sqrt{2}$ (D) 2   (E) 3

13. A knock-out tennis tournament is taking place. There are seven matches (4 quarter finals, 2 semi finals and one final). The results for six of the seven matches are known (but not necessarily in this order):

Bella beats Ann, Celine beats Donna, Gina beats Holly,
Gina beats Celine, Celine beats Bella, Emma beats Farah.

Which result is missing?

(A) Gina beats Bella   (B) Celine beats Ann   (C) Emma beats Celine
(D) Bella beats Holly   (E) Gina beats Emma
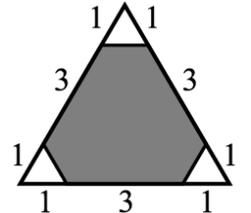
14. What percentage of the area of the triangle is coloured in grey in the adjacent diagram?

(A) 80%   (B) 85%   (C) 88%   (D) 90%   (E) It cannot be calculated.

15. Jilly makes up a multiplication magic square using the numbers 1, 2, 4, 5, 10, 20, 25, 50 and 100. The products of the numbers in each row, column and diagonal should be equal.
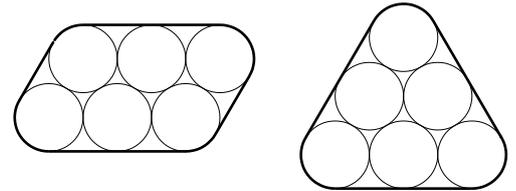In the diagram it can be seen how she has started.
Which number goes into the cell with the question mark?

(A) 2   (B) 4   (C) 5   (D) 10   (E) 25

16. Jack wants to keep six tubes each of diameter 2 cm together using a rubber band. He chooses between the two possible variations shown.
How are the lengths of the rubber bands related to each other?

(A) In the left picture the band is π cm shorter.
(B) In the left picture the band is 4 cm shorter.   (C) In the right picture the band is π cm shorter.
(D) In the right picture the band is 4 cm shorter.   (E) Both bands are equally long.

17. Peter wants to colour in the cells of a 3×3 square so that every row, every column and both diagonals each have three cells with three different colours.
What is the smallest number of colours with which Peter can achieve this?

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7
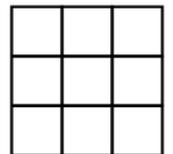
18. Eight cards with the numbers 1, 2, 4, 8, 16, 32, 64, 128 are each in an unmarked envelope. Eva randomly chooses some of these eight envelopes. Ali takes the remaining ones. Both add their numbers together. They find out that Eva's sum is 31 bigger than Ali's sum. How many envelopes has Eva chosen?

(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

19. In the diagram we see a cube and four marked angles.
How big is the sum of those angles?

(A) 315°   (B) 330°   (C) 345°   (D) 360°   (E) 375°
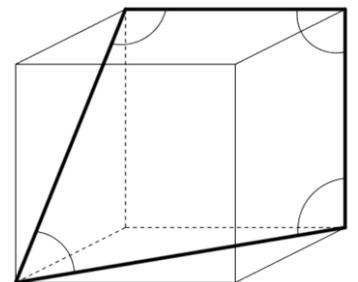
20. In an enclosure there are 2016 kangaroos. Each of them is either red or grey, and there is at least one red and at least one grey kangaroo amongst them.
For each kangaroo K we calculate the fraction obtained, if you take the number of kangaroos of the other colour divided by the kangaroos of the own colour (including K itself).
Determine the sum of these 2016 fractions.

(A) 2016   (B) 1344   (C) 1008   (D) 672   (E) More information is necessary.

21. A creeping plant twists exactly 5 times around a post with circumference 15 cm (as shown in the diagram) and thus reaches a height of 1 m. While the plant grows the height of the plant also grows with constant speed. How long is the creeping plant?
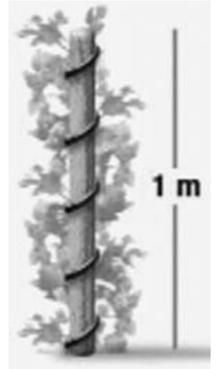
(A) 0.75 m   (B) 1.0 m   (C) 1.25 m   (D) 1.5 m   (E) 1.75 m

22. What is the biggest remainder one can obtain by dividing a two-digit number by the sum of its digits?

(A) 13   (B) 14   (C) 15   (D) 16   (E) 17

23. We consider a 5 × 5 square that is split up into 25 fields. Initially all fields are white. In each move it is allowed to change the colour of two fields that are horizontally or vertically adjacent (i.e. white fields turn black and black ones turn white). What is the smallest number of moves needed to obtain the chessboard colouring shown in the diagram?
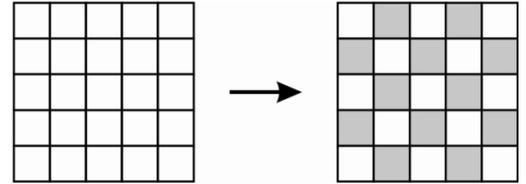
(A) 11   (B) 12   (C) 13   (D) 14   (E) 15

24. A motorboat drives in the middle of a stream. Downstream it needs four hours to get from X to Y. In order to drive back from Y to X it needs six hours. Tree trunks are also floating on the stream.
How many hours does it take for a tree trunk to float in the middle of the stream from X to Y?

(A) 5   (B) 10   (C) 12   (D) 20   (E) 24

25. In the Kangaroo Republic, every month has 40 days, which are numbered through from 1 to 40. Every day with a number that is divisible by 6 is a public holiday, and likewise every day with a prime number.
How often per month does it occur that there is exactly one working day between two public holidays?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

26. Two heights of a triangle have lengths 10 cm and 11 cm.
Which of the following lengths cannot be the length of the third height?

(A) 5 cm   (B) 6 cm   (C) 7 cm   (D) 10 cm   (E) 100 cm

27. Jakob writes down four consecutive positive whole numbers. He calculates all possible sums of three of those numbers und realises that none of those sums is a prime number.
What is the smallest number that Jakob could have written down?

(A) 12   (B) 10   (C) 7   (D) 6   (E) 3

28. Four sportswomen and sportsmen are sitting around a round table for dinner. They do four different sports: ice skating, skiing, hockey and sledging. The person who skies sits to the left of Sandra. The person who ice skates sits opposite Benjamin. Eva and Philipp sit next to each other. A woman sits next to the person who plays hockey.
Which sport does Eva do?

(A) Ice skating   (B) Skiing   (C) Hockey   (D) Sledging
(E) It cannot be determined with this information.

29. A date can be written in the form DD.MM.YYYY; e.g. today's date is 17.03.2016. We call a date "surprising" if all 8 digits used in this notation are different.
In which month does the next surprising date occur?

(A) March   (B) June   (C) July   (D) August   (E) December

30. Exactly 2016 people are taking part in a conference. They are registered as P1 to P2016 in the system. Each person from P1 to P2015 has shaken exactly the amount of other hands that his/her own system number indicates.
How many people did P2016 shake hands with?

(A) 1   (B) 504   (C) 672   (D) 1008   (E) 2015