Kangaroo of Mathematics 2009
Level Student (Grade 11+)
Austria - 23.3.2009

- 3 Points Questions -

1) There are 200 fish in an aquarium. Of which 1% are blue, the rest are yellow. How many yellow fish have to be removed to make the number of blue fish equal 2% of the entire amount of fish?

A) 2 **B)** 4 **C)** 20 **D)** 50 **E)** 100

2) Which of the following numbers is biggest?

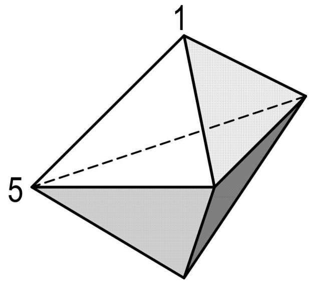
A) $\sqrt{2}-\sqrt{1}$ **B)** $\sqrt{3}-\sqrt{2}$ **C)** $\sqrt{4}-\sqrt{3}$ **D)** $\sqrt{5}-\sqrt{4}$ **E)** $\sqrt{6}-\sqrt{5}$

3) For how many postive whole numbers n is $n^2 + n$ a prime number?
A) 0 **B)** 1 **C)** 2 **D)** for a finite amount more than 2 **E)** for infinite many

4) Mari, Ville and Ossi are going to a coffee shop. Each of them has 3 glasses of juice, 2 cups of ice cream and 5 biscuits. What value could the total bill come up to in the end?

A) ú 39,20 **B)** ú 38,20 **C)** ú 37,20 **D)** ú 36,20 **E)** ú 35,20

5) The diagram on the right shows a solid made up of 6 triangles. Each vertex is assigned a number, two of which are indicated. The total of the three numbers on each triangle is the same. What is the total of all five numbers?
A) 9 **B)** 12 **C)** 17 **D)** 18 **E)** 24



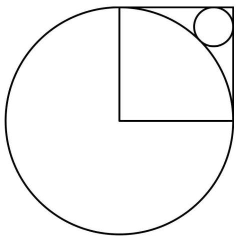
6) The circles k_1 (with centre M_1 and radius 13) and k_2 (with centre M_2 and radius 15) intersect each other in the points P and Q. The length of the distance PQ is 24. What possible value could the distance M_1M_2 be?

A) 2 **B)** 5 **C)** 9 **D)** 14 **E)** 18

7) In a draw there are 2 white, 3 red and 4 blue socks. Lisa knows that one third of the socks have holes but she does not know the colour of the faulty socks. She randomly picks socks from the draw until she has a pair that is useable i.e. she has a pair without holes and of equal colour. What is the minimum amount of socks she has to draw to be certain to get a useable pair?

A) 2 **B)** 3 **C)** 6 **D)** 7 **E)** 8

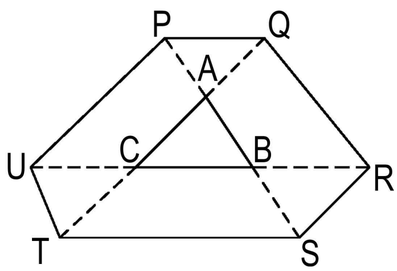
8) The suare in the diagram has side length 1. The radius of the small circle would then be of the length



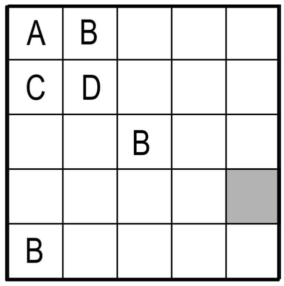
A) $\sqrt{2}-1$ **B)** $\frac{1}{4}$ **C)** $\frac{\sqrt{2}}{4}$ **D)** $1-\frac{\sqrt{2}}{2}$ **E)** $\left(\sqrt{2}-1\right)^2$

9) Each side of a triangle ABC is being extended to the points P, Q, R, S, T and U, so that PA = AB = BS, TC = CA = AQ and UC = CB = BR. The area of ABC is 1. How big is the area of the hexagon PQRSTU?

A) 9 **B)** 10 **C)** 12 **D)** 13 **E)** the value can not be determined for definite



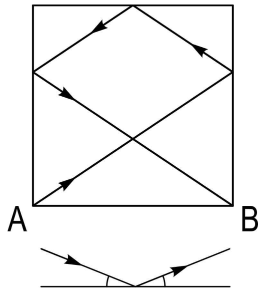
10) In the diagram on the right we want to colour the fields with the colours A, B, C and D so that adjacent fields are always in different colours. (Even fields that share only one corner, count as adjacent.) Some fields have already been coloured in. In which colour can the grey field be coloured in?



A) either A or B **B)** only C **C)** only D
D) either C or D **E)** A, B, C or D

- 4 Points Questions -

11) A (very small) ball is kicked off from point A on a square billiard table with side length 2m. After moving along the shown path and touching the sides three times as indicated, the path ends in point B. How long is the path that the bal travels from A to B? (As indicated on the right: incident angle = emergent angle.)

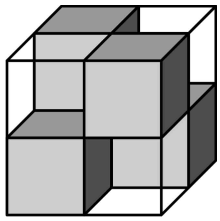


A) 7 **B)** $2\sqrt{13}$ **C)** 8 **D)** $4\sqrt{3}$ **E)** $2\cdot\left(\sqrt{2}+\sqrt{3}\right)$

12) In a group of 2009 kangaroos each one is either light or dark. The smallest of the light kangaroos is bigger than exactly 8 dark kangaroos. One light one is bigger than exactly 9 dark ones, another light one is bigger than exactly 10 dark ones, and so on. Exactly one light cangaroo is bigger than all dark cangaroos. How many light kangaroos are there?

A) 1000 **B)** 1001 **C)** 1002 **D)** 1003 **E)** the situation described is impossible

13) In the diagram to the right a $2 \times 2 \times 2$ cube is made up of four transparent $1 \times 1 \times 1$ cubes and four non-transparent black $1 \times 1 \times 1$ cubes. They are placed in a way so that the entire big cube is non-transparent; i.e. looking at it from the front to the back, the right to the left, the top to the bottom, at no point you can look through the cube. What is the minimum number of black $1 \times 1 \times 1$ cubes needed to make a $3 \times 3 \times 3$ cube non-transparent in the same way?



A) 6 **B)** 9 **C)** 10 **D)** 12 **E)** 18

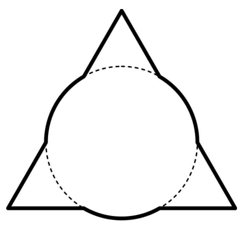
14) On the island of the nobles and liars 25 people are stanging in a queue. The first person in the line claims that everybody behind him is a liar. Each of the other people claims that the person in front of him is a liar. How many liars are actually in the queue? (Nobles are always telling the truth and liars are always lying.)

A) 0 **B)** 12 **C)** 13 **D)** 24 **E)** it cannot be determined

15) Determine the unit digit of the number $1^2 - 2^2 +$ é $- 2008^2 + 2009^2$.

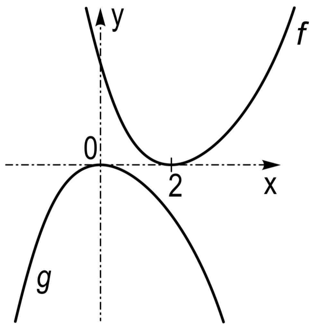
A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

16) An equilateral triangle with side length 3 and a circle with radius 1 have the same centre. What is the perimeter of the figure that is created when the two are being put together?



A) $6+\pi$ **B)** $3+2\pi$ **C)** $9+\frac{\pi}{3}$ **D)** 3π **E)** $9+\pi$

17) The adjacent diagram illustrates the graphs of the two functions f and g. How can we describe the relationship between f and g?

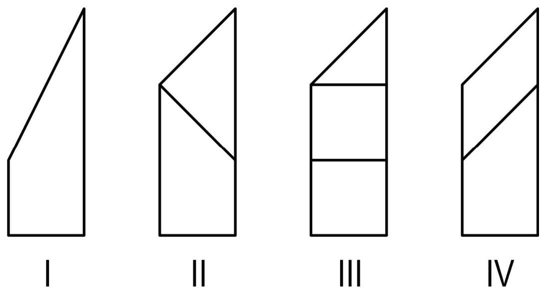


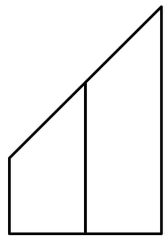
A) $g(x - 2) = -f(x)$ **B)** $g(x) = f(x + 2)$ **C)** $g(x) = -f(-x + 2)$
D) $g(-x) = -f(-x - 2)$ **E)** $g(2 - x) = -f(x)$

18) 100 students take an exam with 4 questions. 90 solve the first question, 85 the second, 80 the third and 70 the fourth. Determine the smallest possible number of students that have solved all four questions.

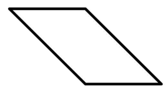
A) 10 **B)** 15 **C)** 20 **D)** 25 **E)** 30

19) In the diagram on the right we see the birdôs-eye view and front elevation of a solid that is defined by flat surfaces (i.e. view from above and the front respectively). Which of the outlines I to IV can be the side elevation (i.e. view from the left) of the same object?





Birdôs-Eye View
(view from above)



Front Elevation
(view from the front)

A) I **B)** II **C)** III **D)** IV **E)** none of them

20) The sum of the number in each line, column and diagonal in the Ămagic squareñ on the right is always constant. Only two numbers are visible. Which number is missing in field a?

a		
		47
	63	

A) 16 **B)** 51 **C)** 54 **D)** 55 **E)** 110

- 5 Points Questions -

21) Two runners each run with constant speed rounds around a racetrack. Both start at the same time at the same point. A is faster than B, takes 3 minutes to cover one lap and overtakes B for the first time after 8 minutes. How long does B take to cover one lap?

A) 6 min **B)** 8 min **C)** 4 min 30 sec **D)** 4 min 48 sec **E)** 4 min 20 sec

22) Let Z be the amount of 8-digit numbers that are made up of all different digits not equal to 0. How many of those number are divisible by 9?

A) $\frac{Z}{8}$ **B)** $\frac{Z}{3}$ **C)** $\frac{Z}{9}$ **D)** $\frac{8Z}{9}$ **E)** $\frac{7Z}{8}$

23) How many 10-digit numbers exist that are solely made up of the numbers 1, 2 and 3 and where adjacent numbers always differ by exactly 1?

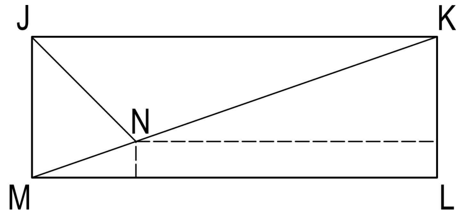
A) 16 **B)** 32 **C)** 64 **D)** 80 **E)** 100

24) For how many whole numbers $n \geq 3$ exists a convex polygon, whose angles are in the ratio 1 : 2 : é : n?

A) 1 **B)** 2 **C)** 3 **D)** 5 **E)** more than 5

25) 55 pupils are taking part in a competition. A jury indicates each question with a Ă+ñ if it is solved correctly, with a Ă–ñ if it is solved incorrectly and a Ă0ñ if it was not attempted. It turns out that no two students had the same amount of Ă+ñ as well as the same amount of Ă–ñ. What is the minimum number of questions that had to be asked in the competition?

A) 6 **B)** 9 **C)** 10 **D)** 11 **E)** 12



26) In a rectangle JKLM the angle bisector in J intersects the diagonal KM in N. The distance of N to LM is 1 and the distance of N to KL is 8.
How long is LM?

A) $8+2\sqrt{2}$ **B)** $11-\sqrt{2}$ **C)** 10 **D)** $8+3\sqrt{2}$ **E)** $11+\frac{\sqrt{2}}{2}$

27) If $k=\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$. How many possible real values exist for k?

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 6

28) The number 1, 2, 3, é , 99 are divided up into n groups. The following rules apply:
¤ Each number is in exactly one group.
¤ There are at least two numbers in each group.
¤ If there are two number in the same group then their sum is not divisible by 3.
Determine the smallest n which fulfills those rules

A) 3 **B)** 9 **C)** 33 **D)** 34 **E)** 66

29) Samantha and her three sisters go to the theater. They have reserved a loge with four seats. Samantha and two of her sisters arrive early and they sit down without paying attention to their seat numbers. Marie arrives later and insists to sit on the seat that is indicated on her ticket. What is the probability that Samantha has to change her seat, if now every sister who has to swap seats insists on sitting on the seat indicated on her ticket.

A) $\frac{3}{4}$ **B)** $\frac{1}{2}$ **C)** $\frac{1}{3}$ **D)** $\frac{1}{4}$ **E)** $\frac{1}{6}$

30) A sequence of whole numbers is defined by $a_0 = 1$, $a_1 = 2$ and $a_{n+2} = a_n + (a_{n+1})^2$ for $n \geq 0$. When a_{2009} is divided by 7 the remainder is

A) 0 **B)** 1 **C)** 2 **D)** 5 **E)** 6